A Memory-Efficient Formulation of the Finite-Difference Time-Domain Method for the Solution of Maxwell Equations

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***Abstract*—With the increase in speed and memory storage in modern computer systems, the finite-difference time-domain (FDTD) method for the solution of electromagnetic problems is rapidly becoming an attractive choice due to its programming simplicity and flexibility in the analysis of a wide range of structures. However, this technique has the drawback of high computer memory requirements and computational power, when analyzing large geometries. In this paper, a modified version of the FDTD method with increased memory efficiency is presented and applied to the calculation of the resonant frequencies of a dielectric resonator coupled to a microstrip line. In this novel approach, the divergence relationship, which spatially links the three electric-field and three magnetic-field components, is used to eliminate one component each of $\mathbf{E}$ and $\mathbf{H}$. This leads to a more memory-efficient formulation, where only four field components are stored in the whole domain, with a direct memory reduction of 33% in the storage of the fields.**

I. Introduction

THE use of the finite-difference time-domain (FDTD) method [1] is very attractive for the electromagnetic analysis of complicated geometries, due mainly to its algorithmic simplicity. However, the computational requirements are high, and computer memory can become a limitation for electrically large bodies. This is due to the fact that the whole domain has to be discretized with rectangular cells having a size of tenths of a wavelength in order to avoid numerical dispersion, which will lead to numerical inaccuracy [2]. The finite-element method [3] is an alternative that allows arbitrary shape discretization, helping to reduce the memory requirement whenever curved structures must be analyzed, but adds to the computation by having to invert very big matrices. The formulation is also more complicated. Spectral-domain methods [6] are the least demanding in terms of memory since only metal regions must be discretized, but they are only able to deal with structures that are laterally infinite. Moreover, the Green's functions [12] that must be used are not always available. Finally, finite-difference methods that rely on higher order space derivatives [4] can significantly reduce storage requirements since they have higher accuracy than FDTD. The formulation though becomes quite complex, especially at the absorbing boundaries, and higher order accuracy cannot be preserved across different dielectric or magnetic materials, where the respective coefficients are discontinuous [2]. Additionally, it has been reported [5] that higher order methods are not as accurate as the Yee algorithms for the simulation of surface waves.

In this paper, we modify the original FDTD formulation to reduce computer memory requirements, allowing a 50% increase in the computational volume for a given computer memory size, with only a moderate increase in computation and code complexity [9]. To our knowledge, only one research group has worked on this problem in the past. In [10], the use of divergence-free electric-field regions was introduced, combined with the scalar-wave equation in order to achieve this goal. In this formulation, the computational domain is subdivided into divergence-free and nonfree regions. Normal FDTD is applied in regions of discontinuities, conductors, sources, and dielectric interfaces, while the scalar-wave equation, requiring four memory elements per cell, instead of the usual six of FDTD, is used in electric-field divergence-free regions. Even though this approach reduces memory requirements, its implementation can be complicated to program due to the necessity of having subregions. More importantly, the memory reduction for this technique is only achieved in some specific geometries, such as planar structures, or other cases where large homogeneous regions exist. Recently, in [7] and [8], a new technique has been proposed, where a 33% memory reduction is achieved for two-dimensional problems with the use of a vector potential formulation. The extension of this technique to three-dimensional problems is under investigation.

In our new method, which we call reduced finite difference time domain (R-FDTD), we eliminate the necessity of subdividing the computational domain into subregions, maintaining the advantage of reducing the number of required field components to four, while also being able to easily treat conductors and source regions. This is achieved by using the divergence-free nature of the electric displacement instead of the electric field, as in [10], and following a specific sequence for the spatial update of the remaining field components. Conductors and source regions are properly treated by calculating the induced charges, which are then used in the divergence of $\mathbf{D}$ ($\nabla \cdot \mathbf{D} = \rho$). In our formulation, although we store only four field components over the

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whole domain and the induced charge wherever conductors are present, we can always reconstruct the two missing field components. This way, standard absorbing boundary conditions, such as Mur [14] or perfect matched layer (PML) [13], can be implemented, as will be shown in Section V. Here lies one significant merit of R-FDTD, i.e., its close resemblance to regular FDTD. For large electromagnetic problems, where memory can really become the decisive factor in using time-domain methods, the transition from FDTD to R-FDTD requires only simple code modifications.

The remainder of this paper is organized as follows. In Section II, we begin with the formulation for divergence-free regions, where no conductors or sources are present, and explain the new algorithm first for the simpler two-dimensional case. Next, we extend R-FDTD to the three-dimensional case and present the algorithm in detail. In Sections III and IV, further extensions are made for the treatment of conductors and source regions. Numerical results are presented in Section V, where the perfect agreement with FDTD confirms the validity of our technique. In Section VI, we conclude with some discussion of the merits and drawbacks of the new method, as well as potential areas of application.

II. New Formulation in Charge-Free Regions

The standard Yee algorithm [1] for the solution of Maxwell equations is based on their discretization in space and time. Starting from system (1), the time marching solution is obtained using a leapfrog scheme [2] to propagate from each component of $\mathbf{E}$ to $\mathbf{H}$ and vice versa as follows:

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{D} &= \nabla \times \mathbf{H} \\ \frac{\partial}{\partial t}\mathbf{B} &= -\nabla \times \mathbf{E}. \end{aligned} \tag{1}$$

In this scheme, all six scalar field components are used explicitly and, therefore, must be stored over the whole computational domain due to the presence of first derivatives in time in (1). In charge-free regions though, these components are not independent, but are linked through the two flux equations. This link can be obtained by taking the divergence on both sides of Ampère's and Faraday's equations of system (1), in which case, we obtain [12]

$$\nabla \cdot \frac{\partial}{\partial t}\mathbf{D} = 0 \tag{2}$$

$$\nabla \cdot \frac{\partial}{\partial t}\mathbf{B} = 0. \tag{3}$$

Upon approximating the time derivatives with central time differences in (2) and with forward time differences in (3), we rewrite the above equations as follows:

$$\nabla \cdot \left(\mathbf{D}^{n+1/2} - \mathbf{D}^{n-1/2}\right) = 0 \tag{4}$$

$$\nabla \cdot \left(\mathbf{B}^{n+1} - \mathbf{B}^{n}\right) = 0. \tag{5}$$

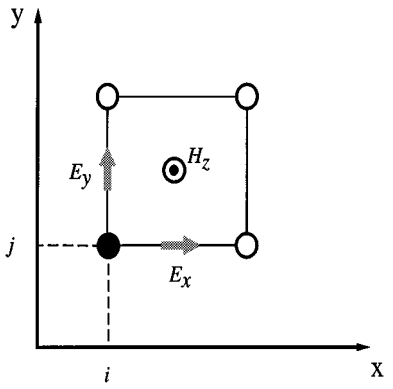

Fig. 1. Definition of the elementary cell for the two-dimensional TE case.

Assuming that initially at time $t = 0 (n = 0)$, all the field components are zero over the whole computational domain, we obtain

$$\nabla \cdot \mathbf{D}^{n+1/2} = 0 \tag{6}$$

$$\nabla \cdot \mathbf{B}^{n+1} = 0. \tag{7}$$

Note here that, the assumption of zero initial fields is generally true everywhere, except the source region, which will be treated in detail in Section IV.

Equations (6) and (7) demonstrate a spatial dependence between the components of vectors $\mathbf{D}$ and $\mathbf{B}$, respectively. Notice that because $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$, this spatial dependence exists regardless of the properties of the medium, which are included in (6) and (7). We use this dependence in (6) and (7) to spatially link two of the field components to the other four, and this way, we reduce the number of field components needed in FDTD from six to four. Similarly, in the two-dimensional case, we reduce the number of components from three to two. In the following, we begin with the simpler two-dimensional case and we continue with the three-dimensional extension. Throughout the paper, we assume that the magnetic permeability coincides with that of free space $(\mu = \mu_0)$. Our results extend directly to mediums with magnetic properties $(\mu \neq \mu_0)$, by a parallel formulation.

A. Two-Dimensional Formulation

Consider at first the TE two-dimensional case, i.e., the only field components are E_x, E_y, and H_z, as shown in Fig. 1. Equation (6) can be used to link E_x and E_y and, upon discretization in space and reordering, can be written as

$$E_y^{n+1/2}(i,j) = \frac{\varepsilon_{i,j-1}}{\varepsilon_{i,j}} E_y^{n+1/2}(i,j-1) - \frac{\Delta y}{\Delta x} \cdot \frac{\varepsilon_{i,j} E_x^{n+1/2}(i,j) - \varepsilon_{i-1,j} E_x^{n+1/2}(i-1,j)}{\varepsilon_{i,j}}. \tag{8}$$

Equation (8) can be incorporated into a standard FDTD algorithm, and upon a proper update scheme, only E_x need to be stored over the whole computational domain. To explain how this is done, assume that $E_x^{n+1/2}$ has already been calculated

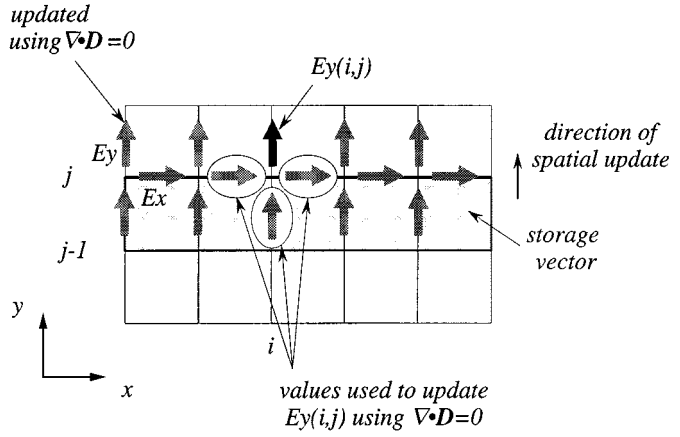

Fig. 2. Spatial-field update mechanism. The storage vector has length N_x and stores $E_y^{n+1/2}(i,\, j-1)$, $i = 1{:}\, N_x$.

with the regular FDTD update equation over the whole computational domain and we want to continue with the update of H_z^{n+1} without having to store $E_y^{n+1/2}$ everywhere. The FDTD update equation for H_z^{n+1} is [1]

$$H_z^{n+1}(i,\, j) = H_z^n(i,\, j) + \frac{\Delta t}{\Delta y \mu_0} \cdot \left[E_x^{n+1/2}(i,\, j+1) - E_x^{n+1/2}(i,\, j)\right] - \frac{\Delta t}{\Delta x \mu_0} \cdot \left[E_y^{n+1/2}(i+1,\, j) - E_y^{n+1/2}(i,\, j)\right]. \quad (9)$$

Therefore, in order to proceed with the $H_z^{n+1}(i,\, j)$ field update, $E_y^{n+1/2}(i,\, j)$ and $E_y^{n+1/2}(i+1,\, j)$ are needed. These can be spatially updated using (8), assuming we have already calculated $E_y^{n+1/2}(i,\, j-1)$ for all i. In other words, the update of $H_z^{n+1}(i,\, j)$ can be done *one j at a time* and for all i, with the prior update of $E_y^{n+1/2}(i,\, j)$ through (8) for all i and having stored only $E_y^{n+1/2}(i,\, j-1)$ for all i. Since $E_y^{n+1/2}(i,\, j-1)$ was needed only for the calculation of $H_z^{n+1}(i,\, j-1)$ and is not used for $H_z^{n+1}(i,\, j)$, $H_z^{n+1}(i,\, j+1)$, …, we can use the same memory locations where we stored $E_y^{n+1/2}(i,\, j-1)$ for all i to store the new values, $E_y^{n+1/2}(i,\, j)$. The spatial update of $E_y^{n+1/2}$ is also illustrated in Fig. 2, where a vector of length N_x (the size of the computational domain along x) is employed for temporary storage of $E_y^{n+1/2}(i,\, j-1)$. Obviously, in order to begin the algorithm, we need to know $E_y^{n+1/2}(i,\, 1)$, which can be obtained with the normal FDTD update equation for E_y as follows:

$$E_y^{n+1/2}(i,\, 1) = E_y^{n-1/2}(i,\, 1) - \frac{\Delta t}{\Delta x \varepsilon_{i,\,1}} \cdot \left[H_z^n(i,\, 1) - H_z^n(i-1,\, 1)\right]. \quad (10)$$

The pseudocode for the TE case of the two-dimensional algorithm is shown at the bottom of the following page.

The pseudocode shows the essence of the algorithm: by updating $H_z(i,\, j)$ one $j = \text{constant}$ each time and for all i, we are able to calculate $E_y(i,\, j)$ only locally for that j and store it in the same vector we stored $E_y(i,\, j-1)$, which is no longer needed. Note that this approach, as opposed to [10], can treat any dielectric discontinuity or inhomogeneity naturally since $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$ for charge-free regions are always divergence-free vectors, while maintaining the generality of the FDTD.

The TM two-dimensional case (i.e., the field components are now H_x, H_y, and E_z) can be formulated along the same lines by using (7) instead of (6). Upon discretization in space and solving, for example, for $H_y^{n+1}(i,\, j)$, (7) is written as (assuming $\mu = \mu_0$)

$$H_y^{n+1}(i,\, j+1) = H_y^{n+1}(i,\, j) - \frac{\Delta y}{\Delta x}\left[H_x^{n+1}(i+1,\, j) - H_x^{n+1}(i,\, j)\right]. \quad (11)$$

Equation (11) can be used in a similar fashion as (8) so that only $H_x(i,\, j)$ needs to be stored everywhere. The formulation is straightforward and will be skipped.

B. Extension to the Three-Dimensional Formulation

Our technique can be extended to the three-dimensional problem by using (6) and (7) simultaneously. Due to spatial dependence of the electric and magnetic fields through these two equations, the total number of variables for the three-dimensional formulation is reduced from six required in regular FDTD to four. In principle, one can independently choose the components of $\mathbf{E}$ and $\mathbf{H}$, which will be only locally updated and not stored, but it will simplify calculations if we choose components of the same direction for $\mathbf{E}$ and $\mathbf{H}$.

Without loss of generality, consider the case where E_y and H_y are the components that are only locally calculated, and are not stored in the whole domain. The update equations for them are derived by discretization of (6)–(7), based on the elementary cell definition shown in Fig. 3, and are as follows:

$$E_y^{n+1/2}(i,\, j,\, k) = \frac{\varepsilon_{i,\,j-1,\,k}}{\varepsilon_{i,\,j,\,k}} E_y^{n+1/2}(i,\, j-1,\, k) - \frac{\Delta y}{\Delta x} \frac{\varepsilon_{i,\,j,\,k} E_x^{n+1/2}(i,\, j,\, k) - \varepsilon_{i-1,\,j,\,k} E_x^{n+1/2}(i-1,\, j,\, k)}{\varepsilon_{i,\,j,\,k}} - \frac{\Delta y}{\Delta z} \frac{\varepsilon_{i,\,j,\,k} E_z^{n+1/2}(i,\, j,\, k) - \varepsilon_{i,\,j,\,k-1} E_z^{n+1/2}(i,\, j,\, k-1)}{\varepsilon_{i,\,j,\,k}} \quad (12)$$

$$H_y^{n+1}(i,\, j+1,\, k) = H_y^{n+1}(i,\, j,\, k) - \frac{\Delta y}{\Delta x}\left[H_x^{n+1}(i+1,\, j,\, k) - H_x^{n+1}(i,\, j,\, k)\right] - \frac{\Delta y}{\Delta z}\left[H_z^{n+1}(i,\, j,\, k+1) - H_z^{n+1}(i,\, j,\, k)\right]. \quad (13)$$

Beginning with the updates of $E_x^{n+1/2}$, $E_z^{n+1/2}$, one realizes that these can be done one $j = \text{constant}$ plane at a time, with the prior spatial update of H_y^n through (13). This is so since, in the FDTD update equations, $E_x^{n+1/2}(i,\, j,\, k)$ depends

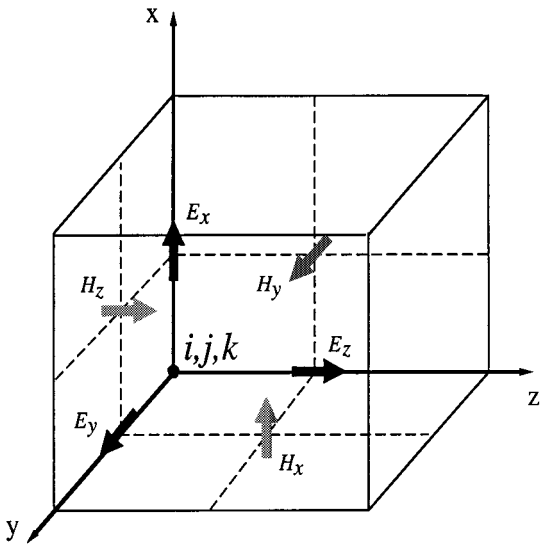

Fig. 3. Definition of the elementary cell for the three-dimensional case.

on $H_y^n(i, j, k)$, $H_y^n(i, j, k-1)$, and $E_z^{n+1/2}(i, j, k)$ depends on $H_y^n(i, j, k)$, $H_y^n(i-1, j, k)$ [1]. In the same way, the updates of H_x^{n+1}, H_z^{n+1} can also be done one $j = \text{constant plane}$ at a time, with the prior spatial update of $E_y^{n+1/2}$ through (12).

As in the two-dimensional formulation, a storage of the y components of the fields at the previous location along y is required. For the three-dimensional case, this means that a two-dimensional array having size of $N_x \times N_z$ must be used for each one of the removed components. The extra memory requirement for these arrays is not significant, and since the choice of which component to remove is arbitrary, we can always choose to remove the field components in the direction for which the corresponding arrays are minimized. The pseudocode for the three-dimensional algorithm is shown at the bottom of page 1315.

Again note the innovative feature of the formulation: e.g., H_y^n is calculated in the **E**-field update part of the algorithm, one $j = \text{constant plane}$ at a time and is stored in the same memory locations as for the previous j. Correspondingly, $E_y^{n+1/2}$ is calculated in the **H**-field update part, also one $j = \text{constant plane}$ at a time and is also stored "in place."

As in the FDTD formulation, the stability condition for this new numerical technique is imposed by the Courant condition [2], which requires that, in one time step, the propagating wave must not travel more than one computational cell.

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1) E_x field update. Use regular FDTD:
   for i = 1: $N_x - 1$
      for j = 2: $N_y - 1$

$$E_x^{n+1/2}(i, j) = E_x^{n-1/2}(i, j) + \frac{\Delta t}{\Delta y \varepsilon_{i,j}} [H_z^n(i, j) - H_z^n(i, j-1)] \qquad \text{(FDTD)}$$

      end j
   end i
2) H_z field update.
   1) Get $E_y^{n+1/2}(i, 1)$ from regular FDTD and store it in $v_E(i)$ [eq. (10)].
   2) Update H_z for all i and j:
      for j = 1: $N_y - 1$
         for i = 1: $N_x - 1$

$$H_z^{n+1}(i, j) = H_z^n(i, j) + \frac{\Delta t}{\Delta y \mu_0} \left[E_x^{n+1/2}(i, j+1) - E_x^{n+1/2}(i, j)\right] + - \frac{\Delta t}{\Delta x \mu_0} [v_E(i+1) - v_E(i)] \qquad \text{(FDTD)}$$

         end i
         Update spatially $E_y^{n+1/2}(i, j+1)$ for next j iteration, [eq. (8)]:
         for i = 2: $N_x - 1$

$$v_E(i) = \frac{\varepsilon_{i,j}}{\varepsilon_{i,j+1}} v_E(i) - \frac{\Delta y}{\Delta x \varepsilon_{i,j+1}} \left[\varepsilon_{i,j+1} E_x^{n+1/2}(i, j+1) + - \varepsilon_{i-1,j+1} E_x^{n+1/2}(i-1, j+1)\right] \qquad (\nabla \cdot \mathbf{D} = 0)$$

         end i
      end j

III. Conductor Treatment

In the presence of conductors, the divergence of the electric flux is not zero anymore, but rather equals the induced charge on the conductor ρ,[1] and (6) is not satisfied. In order to use our method in the presence of a conductor, the induced charge must be found. Indeed, in the following, we proceed to show how this can be done through the use of the regular FDTD update equations, independently of our new algorithm, and then we discuss the extension of R-FDTD to include regions with conductors. Although we treat electric conductors in detail, all the discussions extend easily to the case of magnetic conductors with a parallel analysis.

Revisiting (4), let us denote by $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ the difference approximation to $\nabla\cdot\mathbf{D}^{n+1/2}$. Based on the elementary cell definition shown in Fig. 3, $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ can be written as

$$\begin{aligned}\Delta\mathbf{D}^{n+1/2}&(i, j, k)\\ &= \frac{\varepsilon_{i,j,k}E_x^{n+1/2}(i, j, k) - \varepsilon_{i-1,j,k}E_x^{n+1/2}(i-1, j, k)}{\Delta x}\\ &\quad + \frac{\varepsilon_{i,j,k}E_y^{n+1/2}(i, j, k) - \varepsilon_{i,j-1,k}E_y^{n+1/2}(i, j-1, k)}{\Delta y}\\ &\quad + \frac{\varepsilon_{i,j,k}E_z^{n+1/2}(i, j, k) - \varepsilon_{i,j,k-1}E_z^{n+1/2}(i, j, k-1)}{\Delta z}.\end{aligned} \tag{14}$$

Equation (4) in discrete-space discrete-time form will then be written as

$$\Delta\mathbf{D}^{n+1/2}(i, j, k) = \Delta\mathbf{D}^{n-1/2}(i, j, k). \tag{15}$$

It is crucial for the rest of the development to realize here that (15) is always satisfied after E_x, E_y, and E_z have been updated from the usual FDTD equations since these are naturally divergence free [this can also be shown by direct substitution of the regular FDTD $\mathbf{E}$-field update equations in (14)]. In the absence of conductors or sources, $\Delta\mathbf{D}^{n+1/2}(i, j, k) = 0$, which is nothing more than the discretized version of (6).

Now, to fix ideas, assume that there is a conductor segment at cell (i_0, j_0, k_0), extending on the xz-plane. That means that $E_x(i_0, j_0, k_0)$ and $E_z(i_0, j_0, k_0)$ are always zero. We will examine in detail the effect of setting $E_x(i_0, j_0, k_0)$ to zero, the results for $E_z(i_0, j_0, k_0) = 0$ follow by a simple rotation of coordinates. In regular FDTD, $E_x(i_0, j_0, k_0)$ is first updated with the usual update equation, as for all the other (i, j, k), and later set to zero per the following steps.

Step 1) Usual FDTD update

$$\begin{aligned}E_x^{n+1/2}&(i_0, j_0, k_0)\\ &= E_x^{n-1/2}(i_0, j_0, k_0) + \frac{\Delta t}{\Delta y\varepsilon_{i_0,j_0,k_0}}\\ &\quad\cdot\Big[H_z^n(i_0, j_0, k_0) - H_z^n(i_0, j_0-1, k_0)\Big] - \frac{\Delta t}{\Delta z\varepsilon_{i_0,j_0,k_0}}\\ &\quad\cdot\Big[H_y^n(i_0, j_0, k_0) - H_y^n(i_0, j_0, k_0-1)\Big].\end{aligned} \tag{16}$$

Note here that, from the previous time update $E_x^{n-1/2}(i_0, j_0, k_0) = 0$ and, therefore, $E_x^{n+1/2}(i_0, j_0, k_0)$ depends only on the magnetic field. Call $E_{0x}^{n+1/2}(i_0, j_0, k_0)$ this temporary value of $E_x^{n+1/2}(i_0, j_0, k_0)$.

Step 2) Later in the code, $E_x^{n+1/2}(i_0, j_0, k_0) = 0$.

Now, notice that after Step 1), but before Step 2), $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ satisfies (15) for all (i, j, k) since only the regular FDTD equations have been involved before Step 2). After Step 2), $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ will be affected at cells (i_0, j_0, k_0) and (i_0+1, j_0, k_0) [i.e., at the two ends of vector $E_x^{n+1/2}(i_0, j_0, k_0)$], as can be seen by (14). Specifically, bringing to the right-hand side of (15) the term to be zeroed in Step 2), we can write

$$\begin{aligned}\Delta\mathbf{D}^{n+1/2}(i_0, j_0, k_0) &= \Delta\mathbf{D}^{n-1/2}(i_0, j_0, k_0)\\ &\quad - \frac{\varepsilon_{i_0,j_0,k_0}}{\Delta x}E_{0x}^{n+1/2}(i_0, j_0, k_0)\end{aligned} \tag{17}$$

$$\begin{aligned}\Delta\mathbf{D}^{n+1/2}(i_0+1, j_0, k_0) &= \Delta\mathbf{D}^{n-1/2}(i_0+1, j_0, k_0)\\ &\quad + \frac{\varepsilon_{i_0,j_0,k_0}}{\Delta x}E_{0x}^{n+1/2}(i_0, j_0, k_0).\end{aligned} \tag{18}$$

Equations (17) and (18) provide recursive relations to calculate $\Delta\mathbf{D}^{n+1/2}(i_0, j_0, k_0)$ and $\Delta\mathbf{D}^{n+1/2}(i_0+1, j_0, k_0)$, beginning at $n = 0$ with $\Delta\mathbf{D}^{-1/2}(i_0, j_0, k_0) = \Delta\mathbf{D}^{-1/2}(i_0+1, j_0, k_0) = 0$ and $E_{0x}^{1/2} = 0$. The obvious solutions are

$$\Delta\mathbf{D}^{n+1/2}(i_0, j_0, k_0) = -\frac{\varepsilon_{i_0,j_0,k_0}}{\Delta x}\sum_{m=1}^{n}E_{0x}^{m+1/2}(i_0, j_0, k_0) \tag{19}$$

$$\Delta\mathbf{D}^{n+1/2}(i_0+1, j_0, k_0) = \frac{\varepsilon_{i_0,j_0,k_0}}{\Delta x}\sum_{m=1}^{n}E_{0x}^{m+1/2}(i_0, j_0, k_0) \tag{20}$$

Since $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ is just the discrete-time discrete-space approximation to $\nabla\cdot\mathbf{D}$, it is clear that the imposition of $E_x(i_0, j_0, k_0) = 0$ created induced charges at cells (i_0, j_0, k_0) and (i_0+1, j_0, k_0), which are given by the right-hand side of (19) and (20). The induced charges resulting from $E_z(i_0, j_0, k_0) = 0$ can be obtained by (19) and (20) by interchanging the x- for z-coordinate; consequently, $\Delta\mathbf{D}^{n+1/2}(i, j, k)$ due *only* to $E_z(i_0, j_0, k_0) = 0$ will be nonzero at cells (i_0, j_0, k_0) and (i_0, j_0, k_0+1). At these cells, the following equations hold:

$$\Delta\mathbf{D}^{n+1/2}(i_0, j_0, k_0) = -\frac{\varepsilon_{i_0,j_0,k_0}}{\Delta z}\sum_{m=1}^{n}E_{0z}^{m+1/2}(i_0, j_0, k_0) \tag{21}$$

$$\Delta\mathbf{D}^{n+1/2}(i_0, j_0, k_0+1) = \frac{\varepsilon_{i_0,j_0,k_0}}{\Delta z}\sum_{m=1}^{n}E_{0z}^{m+1/2}(i_0, j_0, k_0). \tag{22}$$

It is obvious now that the charge at cell (i_0, j_0, k_0) due to both E_x and E_z being zero, will actually be the sum of the right-hand sides of (19) and (21).

Now, we can introduce a new variable $\rho(i, j, k)$, for the PEC cells only, which represents the induced charge of that cell. In the example analyzed above, (19) implies that the charge induced to cell (i_0, j_0, k_0), due to the imposition

[1]The induced charge on a perfect electric conductor (PEC) is, in reality, a surface charge. However, due to the discretization in the FDTD, it will appear as a volume charge, dispersed inside the cells comprising the conductor. An equivalent surface charge can always be obtained by multiplying ρ by the size of the cell in the direction normal to the surface of the conductor.

of $E_x^{n+1/2}(i_0, j_0, k_0) = 0$, will be updated based on the following equation:

$$\rho^{n+1/2}(i_0, j_0, k_0) = \rho^{n-1/2}(i_0, j_0, k_0) - \frac{\varepsilon_{i_0, j_0, k_0}}{\Delta x} E_{0x}^{n+1/2}(i_0, j_0, k_0) \tag{23}$$

while, from (20), the charge induced to cell (i_0+1, j_0, k_0) and due to $E_x^{n+1/2}(i_0, j_0, k_0) = 0$ will be updated by

$$\rho^{n+1/2}(i_0+1, j_0, k_0) = \rho^{n-1/2}(i_0+1, j_0, k_0) + \frac{\varepsilon_{i_0, j_0, k_0}}{\Delta x} E_{0x}^{n+1/2}(i_0, j_0, k_0). \tag{24}$$

---

1) E_x, E_z field updates.

a) Get $H_y^n(i, 2, k)$ from regular FDTD and store it in $v_H(i, k)$.

b) Update E_x and E_z for all i, k and j:

for j $= 2$: $N_y - 1$

for i $= 1$: $N_x - 1$, k $= 1$: $N_z - 1$

$$E_x^{n+1/2}(i, j, k) = E_x^{n-1/2}(i, j, k) + \frac{\Delta t}{\Delta y \varepsilon_{i, j, k}} [H_z^n(i, j, k) - H_z^n(i, j-1, k)] + - \frac{\Delta t}{\Delta z \varepsilon_{i, j, k}} [v_H(i, k) - v_H(i, k-1)] \qquad \text{(FDTD)}$$

$$E_z^{n+1/2}(i, j, k) = E_z^{n-1/2}(i, j, k) + \frac{\Delta t}{\Delta x \varepsilon_{i, j, k}} [v_H(i, k) - v_H(i-1, k)] + - \frac{\Delta t}{\Delta y \varepsilon_{i, j, k}} [H_x^n(i, j, k) - H_x^n(i, j-1, k)] \qquad \text{(FDTD)}$$

end k, i

Update spatially $H_y^n(i, j+1, k)$ for next j iteration [eq. (13)]:

for i $= 2$: $N_x - 1$, k $= 2$: $N_z - 1$

$$v_H(i, k) = v_H(i, k) - \frac{\Delta y}{\Delta x} [H_x^n(i+1, j, k) - H_x^n(i, j, k)] + - \frac{\Delta y}{\Delta z} [H_z^n(i, j, k+1) - H_z^n(i, j, k)] \qquad (\nabla \cdot \mathbf{H} = 0)$$

end k, i

end j

2 H_x, H_z field updates.

a) Get $E_y^{n+1/2}(i, 1, k)$ from regular FDTD and store it in $v_E(i, k)$.

b) Update H_x and H_z for all i, k and j:

for j $= 1$: $N_y - 1$

for i $= 1$: $N_x - 1$, k $= 1$: $N_z - 1$

$$H_x^{n+1}(i, j, k) = H_x^n(i, j, k) + \frac{\Delta t}{\Delta z \mu_0} [v_E(i, k+1) - v_E(i, k)] + - \frac{\Delta t}{\Delta y \mu_0} \left[E_z^{n+1/2}(i, j+1, k) - E_z^{n+1/2}(i, j, k)\right] \qquad \text{(FDTD)}$$

$$H_z^{n+1}(i, j, k) = H_z^n(i, j, k) + \frac{\Delta t}{\Delta y \mu_0} \left[E_x^{n+1/2}(i, j+1, k) - E_x^{n+1/2}(i, j, k)\right] + - \frac{\Delta t}{\Delta x \mu_0} [v_E(i+1, k) - v_E(i, k)] \qquad \text{(FDTD)}$$

end k, i

Update spatially $E_y^{n+1/2}(i, j+1, k)$ for next j iteration [eq. (12)]:

for i $= 2$: $N_x - 1$, k $= 2$: $N_z - 1$

$$v_E(i, k) = \frac{\varepsilon_{i, j, k}}{\varepsilon_{i, j+1, k}} v_E(i, k) - \frac{\Delta y}{\Delta x \varepsilon_{i, j+1, k}} \left[\varepsilon_{i, j+1, k} E_x^{n+1/2}(i, j+1, k) + - \varepsilon_{i-1, j+1, k} E_x^{n+1/2}(i-1, j+1, k)\right] - \frac{\Delta y}{\Delta z \varepsilon_{i, j+1, k}} \left[\varepsilon_{i, j+1, k} E_z^{n+1/2}(i, j+1, k) + - \varepsilon_{i, j+1, k-1} E_z^{n+1/2}(i, j+1, k-1)\right] \qquad (\nabla \cdot \mathbf{D} = 0)$$

end k, i

end j

Similar equations can be written for the charges induced by $E_z^{n+1/2}(i_0, j_0, k_0) = 0$. Two remarks are in order here: first, observe that $E_{0x}^{n+1/2}(i_0, j_0, k_0)$ is nothing else, but $E_x^{n+1/2}(i_0, j_0, k_0)$, as calculated in Step 1) [we just called it a different name so that it does not get confused with the zero value of $E_x^{n+1/2}(i_0, j_0, k_0)$ of Step 2)]. This quantity is calculated within the regular FDTD algorithm and, therefore, it is already available to us; second, (23) and (24) do not give the *total* induced charge in cells (i_0, j_0, k_0) and (i_0+1, j_0, k_0), but only the part due to $E_x^{n+1/2}(i_0, j_0, k_0) = 0$. If the PEC extends to cell $(i_0 - 1, j_0, k_0)$, for example, there will be another contribution to cell (i_0, j_0, k_0) from $E_x^{n+1/2}(i_0 - 1, j_0, k_0) = 0$, as implied by (20). Equation (23) will then become

$$\rho^{n+1/2}(i_0, j_0, k_0) = \rho^{n-1/2}(i_0, j_0, k_0) + \frac{1}{\Delta x} \cdot \left[\varepsilon_{i_0-1, j_0, k_0} E_{0x}^{n+1/2}(i_0 - 1, j_0, k_0) - \varepsilon_{i_0, j_0, k_0} E_{0x}^{n+1/2}(i_0, j_0, k_0)\right]. \tag{25}$$

As a concrete example of the charge calculation, consider a conductor plate on the xz-plane at $j = j_c$ and extending for $i_0 \le i \le i_1, k_0 \le k \le k_1$. After having updated $E_x^{n+1/2}(i, j, k)$ and $E_z^{n+1/2}(i, j, k)$ everywhere, we update the charge and then set the tangential fields on the conductor to zero, as shown in the algorithm at the bottom of this page.

Note from this example that the charges on the edges of the conductor are updated differently from the charges inside the conductor due to the fact that the field components normal to the edges, but external to the conductor, are not zero. Based on this example, the generalization for PECs of any orientation should be obvious.

Having found the charge in an independent way, as explained above, we can now use it to extend the reduced FDTD algorithm to include electric conductors. Of course, we need to introduce a new variable, i.e., the charge, for the conductor cells. The number of conductor cells though is usually very small with respect to the total number of cells in the whole domain and, therefore, there is practically a negligible increase in memory requirements. The charge is updated using the appropriate equation between (23)–(25) and along the lines of the example described above. One problem arises when a conductor extends in the direction of that electric-field component that is spatially updated. For example, in the case we update spatially E_y and H_y, and a conductor lies along the y-direction, we cannot use equations like (23)–(25) since we do not have $E_{0y}^{n+1/2}(i_0, j_0, k_0)$ (which is $E_y^{n+1/2}(i_0, j_0, k_0)$ before being set to zero). However, since $E_y^{n-1/2}(i_0, j_0, k_0) = 0$, $E_y^{n+1/2}(i_0, j_0, k_0)$ will

---

1) Charges due to E_x and setting $E_x^{n+1/2}$ to zero:

for $k = k_0$: k_1,

$$\rho^{n+1/2}(i_0, j_c, k) = \rho^{n-1/2}(i_0, j_c, k) - \frac{\varepsilon_{i_0, j_c, k}}{\Delta x} E_x^{n+1/2}(i_0, j_c, k)$$

$$\rho^{n+1/2}(i_1, j_c, k) = \rho^{n-1/2}(i_1, j_c, k) + \frac{\varepsilon_{i_1-1, j_c, k}}{\Delta x} E_x^{n+1/2}(i_1 - 1, j_c, k)$$

for $i = i_0 + 1$: $i_1 - 1$,

$$\rho^{n+1/2}(i, j_c, k) = \rho^{n-1/2}(i, j_c, k) + \frac{1}{\Delta x}\left[\varepsilon_{i-1, j_c, k} E_x^{n+1/2}(i - 1, j_c, k) - \varepsilon_{i, j_c, k} E_x^{n+1/2}(i, j_c, k)\right]$$

$$E_x^{n+1/2}(i - 1, j_c, k) = 0$$

end i

$$E_x^{n+1/2}(i_1 - 1, j_c, k) = 0$$

end k

2) Charges due to E_z and setting $E_z^{n+1/2}$ to zero:

for $i = i_0$: i_1,

$$\rho^{n+1/2}(i, j_c, k_0) = \rho^{n-1/2}(i, j_c, k_0) - \frac{\varepsilon_{i, j_c, k_0}}{\Delta z} E_z^{n+1/2}(i, j_c, k_0)$$

$$\rho^{n+1/2}(i, j_c, k_1) = \rho^{n-1/2}(i, j_c, k_1) + \frac{\varepsilon_{i, j_c, k_1-1}}{\Delta z} E_x^{n+1/2}(i, j_c, k_1 - 1)$$

for $k = k_0 + 1$: $k_1 - 1$,

$$\rho^{n+1/2}(i, j_c, k) = \rho^{n-1/2}(i, j_c, k) + \frac{1}{\Delta z}\left[\varepsilon_{i, j_c, k-1} E_z^{n+1/2}(i, j_c, k - 1) - \varepsilon_{i, j_c, k} E_z^{n+1/2}(i, j_c, k)\right]$$

$$E_z^{n+1/2}(i, j_c, k - 1) = 0$$

end k

$$E_z^{n+1/2}(i, j_c, k_1 - 1) = 0$$

end i

only depend on the magnetic field, and the regular FDTD update would be

$$E_y^{n+1/2}(i_0, j_0, k_0) = \frac{\Delta t}{\Delta z \varepsilon_{i_0, j_0, k_0}} \Big[H_x^n(i_0, j_0, k_0) - H_x^n(i_0, j_0, k_0 - 1)\Big] - \frac{\Delta t}{\Delta x \varepsilon_{i_0, j_0, k_0}} \Big[H_z^n(i_0, j_0, k_0) - H_z^n(i_0 - 1, j_0, k_0)\Big]. \tag{26}$$

Since we have H_x^n and H_z^n everywhere, we can use the right-hand side of (26) in the place of $E_{0y}^{n+1/2}(i_0, j_0, k_0)$ to update the charge for conductors that extend in the y-direction.

The charge ρ found in the way exhibited above is simply added to the left-hand side of (15) for the conductor cells. Observe also that we do not have to set E_y explicitly to zero whenever it is tangential to a PEC, it will be forced to zero (i.e., to a very small numerical value) by (12) with the addition of ρ.

As a final remark, we would like to note that (23)–(25) are nothing more than discrete forms of the differential equation of the conservation of charge, namely,

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J} \tag{27}$$

where $\mathbf{J}$ is the induced current.[2] Indeed, consider Ampère's law

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}. \tag{28}$$

Across a conductor, the tangential electric-field components are zero and, therefore,

$$\mathbf{J} = (\nabla \times \mathbf{H})_t \tag{29}$$

where subscript t refers to the tangential components. Therefore, (27) becomes

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\nabla \times \mathbf{H})_t = \nabla \cdot (\nabla \times \mathbf{H})_n \tag{30}$$

where n denote the normal to the conductor surface component of $\nabla \times \mathbf{H}$. Equations (23)–(25) can be derived anew from the discretization of (30). We preferred to show how they can be obtained directly from FDTD in order to demonstrate their actual application and to avoid technical difficulties with the edges of the conductors.

IV. Source Treatment

The standard way to excite the electromagnetic field in FDTD is to impose a time-varying field distribution in a certain space region. As can be expected, $\mathbf{D}$ does not maintain its divergence-free property, though in the case of homogeneous media, this happens only for these cells where the imposed electric flux is terminated. In any case, we will be able to compute the appropriate correction term in advance (i.e. before the start of the time iteration) and use it in $\nabla \cdot \mathbf{D}$. The analysis follows the same lines as for the conductor treatment, i.e., finding a recursive relation for $\Delta \mathbf{D}^{n+1/2}$ (which is, again, the discrete-time discrete-space approximation to $\nabla \cdot \mathbf{D}$) and solving it to obtain the equivalent charge induced by the source. For example, suppose a soft electric-field source along x is imposed on the plane $k = k_0$, for $i_0 \le i \le i_1$, $j_0 \le j \le j_1$, as shown in Fig. 4

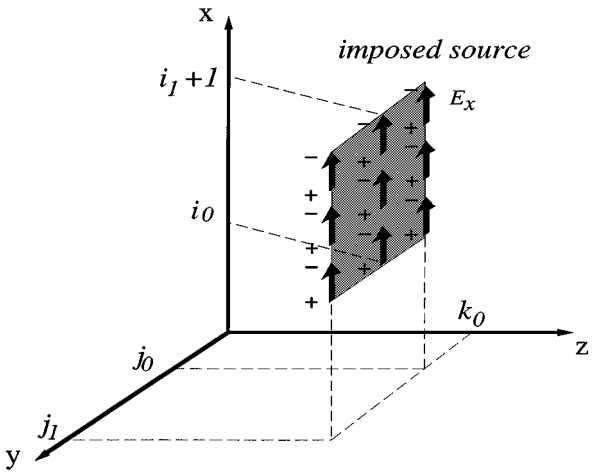

Fig. 4. Imposed source field on a portion of the xy-plane.

$$E_x^{n+1/2}(i, j, k_0) = E_x^{n+1/2}(i, j, k_0) + \exp\left(-\frac{(n - n_0)\Delta t}{T}\right)^2, \quad \begin{matrix} i_0 \le i \le i_1 \\ j_0 \le j \le j_1. \end{matrix} \tag{31}$$

We now need to determine the equation satisfied by $\Delta \mathbf{D}^{n+1/2}$ in the source region so that we can use it in our algorithm for the spatial update of one electric-field component. Just before the application of the source in the code, $\Delta \mathbf{D}^{n+1/2}$ satisfies, as usual, (15). It is straightforward to show that, after imposing the soft source according to (31), the following equations hold:

1) For $i_0 < i \le i_1$, $j_0 \le j \le j_1$

$$\Delta \mathbf{D}^{n+1/2}(i, j, k_0) = \Delta \mathbf{D}^{n-1/2}(i, j, k_0) + \frac{\varepsilon_{i, j, k_0} - \varepsilon_{i-1, j, k_0}}{\Delta x} \cdot \exp\left(-\frac{(n - n_0)\Delta t}{T}\right)^2. \tag{32}$$

2) For $i = i_0$, $j_0 \le j \le j_1$

$$\Delta \mathbf{D}^{n+1/2}(i, j, k_0) = \Delta \mathbf{D}^{n-1/2}(i, j, k_0) + \frac{\varepsilon_{i_0, j, k_0}}{\Delta x} \exp\left(-\frac{(n - n_0)\Delta t}{T}\right)^2 \tag{33}$$

since there is no imposed field in $E_x^{n+1/2}(i_0 - 1, j, k_0)$.

3) For $i = i_1 + 1$, $j_0 \le j \le j_1$

$$\Delta \mathbf{D}^{n+1/2}(i, j, k_0) = \Delta \mathbf{D}^{n-1/2}(i, j, k_0) - \frac{\varepsilon_{i_1, j, k_0}}{\Delta x} \exp\left(-\frac{(n - n_0)\Delta t}{T}\right)^2 \tag{34}$$

since there is no imposed field in $E_x^{n+1/2}(i_1 + 1, j, k_0)$.

For homogeneous media, where ε is constant, (32) implies that $\Delta \mathbf{D}^{n+1/2}(i, j, k_0) = 0$ for $i_0 < i \le i_1$, $j_0 \le j \le j_1$. That

[2]Again, here, the induced current on the conductor appears as a volume current. For the case of a conductor on the xz-plane, an equivalent surface current can be obtained by multiplying $\mathbf{J}$ by Δy.

is, $\Delta\mathbf{D}^{n+1/2}(i, j, k_0)$ is nonzero only where the imposed electric flux is terminated, i.e., for $i = i_0$ and $i = i_1 + 1$, as shown by (33) and (34) (see also Fig. 4). In the general case, we can use the recursive equations (32)–(34) with the initial condition $\Delta\mathbf{D}^{-1/2} = 0$ to write the following equation for $\Delta\mathbf{D}^{n+1/2}$, which holds for all $j_0 \le j \le j_1$, $k = k_0$, and the i specified:

$$\Delta\mathbf{D}^{n+1/2}(i, j, k_0) = \begin{cases} \dfrac{\varepsilon_{i,j,k_0} - \varepsilon_{i-1,j,k_0}}{\Delta x} \displaystyle\sum_{m=1}^{n} \exp\left(-\frac{(m-n_0)\Delta t}{T}\right)^2, \\ \quad \text{if } i_0 < i \le i_1 \\ \dfrac{\varepsilon_{i_0,j,k_0}}{\Delta x} \displaystyle\sum_{m=1}^{n} \cdot \exp\left(-\frac{(m-n_0)\Delta t}{T}\right)^2, \\ \quad \text{if } i = i_0 \\ -\dfrac{\varepsilon_{i_1,j,k_0}}{\Delta x} \displaystyle\sum_{m=1}^{n} \exp\left(-\frac{(m-n_0)\Delta t}{T}\right)^2, \\ \quad \text{if } i = i_1 + 1. \end{cases} \tag{35}$$

The terms on the right-hand side of (35) can be calculated for each n before the beginning of the time update of the fields, and then used at the appropriate time iteration. In the most common case where the source is imposed in a homogeneous medium, the right-hand side of (35) simplifies greatly and one needs to store only one of the two lower terms for each n since the first term is zero and the other two are just opposite.

Finally, note that, for homogeneous media, the fact that the divergence of $\mathbf{D}$ is nonzero only wherever the imposed field flux is terminated can be explained by equivalent charges that are induced at these points, as can be seen in Fig. 4. In the intermediate cells of the source region, the charges cancel each other, but not so at the termination of the imposed E_x-field. The charges induced by the source keep accumulating according to (35).

V. Numerical Results

As a validation of the new formulation, a microstrip coupled dielectric resonator is analyzed with the R-FDTD and FDTD methods and results are compared with measured data. Perfect agreement between FDTD and R-FDTD is observed, indicating the equivalence of the two methods. Good agreement with the measured data is also observed.

The measurement was performed on a cylindrical resonator of 2.26-mm diameter and 0.91-mm height, made of a Perovskite based on Ba, Zn, Ta–oxide (see Fig. 5). The dielectric resonator (manufactured by Trans-Tech, model D8733-0089-036) has a relative dielectric permittivity of 30.15, and a Q factor at 10 GHz of 12 200. The resonator is placed on top of a dielectric substrate, the substrate is made of 124-μm-thick alumina (relative dielectric constant $\varepsilon_r = 9.8$) with a ground metal (gold plated) on the other side, as shown in Fig. 5. A 9-mm-long offset conductor of 124 –μm width is used to couple the field to the resonator. The whole substrate was enclosed on a rectangular waveguide whose dimensions are 2.5 × 4.6× 7.5 mm, to reduce radiation losses (see Fig. 5). The resonator was placed directly on the substrate (no spacer was used) with one edge exactly at

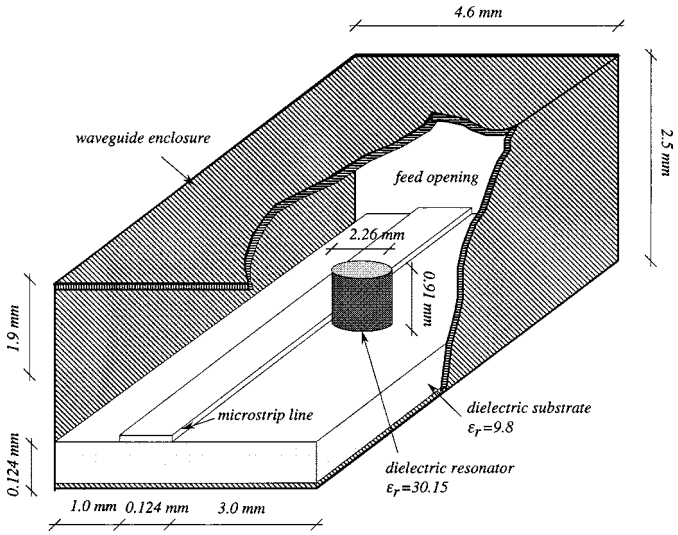

Fig. 5. Geometry of the microstrip coupled dielectric resonator analyzed.

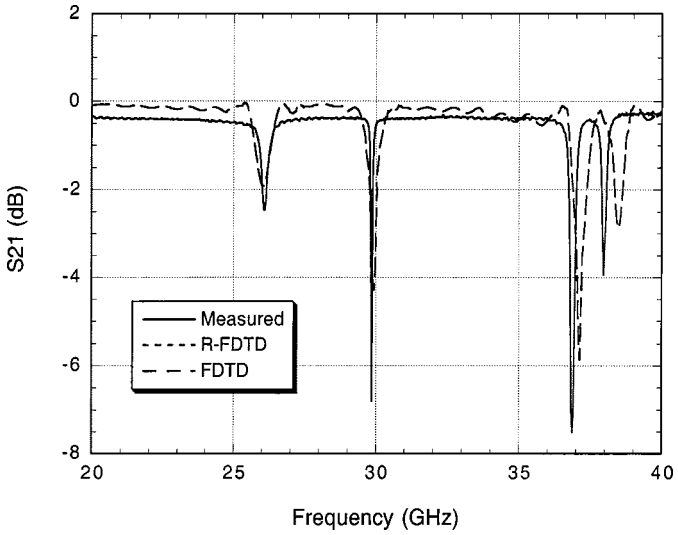

Fig. 6. Comparison of measured and simulated magnitude of the transmission coefficient for the microstrip coupled dielectric resonator.

the edge of the microstrip line (no overlapping). The measurement of the resonance frequency was done using a network analyzer (HP 8510C) with coaxial probes (Picoprobe Model 40 A). Wraparound-type ground transitions for the probes were done with silver epoxy. By using the thru-reflect-line (TRL) [15] calibration technique, 25-dB return-loss sensitivity was achieved over the band going from 20 to 40 GHz. The total length of the microstrip from the two reference planes was 7.5 mm. The measured transmission coefficient for the proposed structure is reported in Fig. 6. The desired resonance frequency is obtained at 29.862 GHz, and a box resonance is also observed at 26.075 GHz and two higher resonances, respectively, at 36.875 and 37.963 GHz.

To simulate the described structure, the whole domain was discretized with a uniform grid having the dimensions of 100 × 60× 166, respectively, along x-, y-, and z-directions. The corresponding grid size was $\Delta x = 45.2$ μm, $\Delta y = 41.33$ μm, and $\Delta z = 45.2$ μm; this choice allows to best fit the resonator

height and substrate dimensions with the grid. The excitation is obtained through a Gaussian pulse under the strip conductor having a width of $T = 10$ ps, corresponding to a maximum frequency of 50 GHz. The number of time steps was set at 10 000. Reflection conditions are used on the metal sidewalls of the waveguide, and PMLs [13] are used at the two waveguide ends. All field components are calculated and stored inside the PML. A standard procedure is used to extract the transmission parameters [16] for the analyzed structure. The transmission coefficient obtained is shown in Fig. 6 for normal FDTD and our formulation (R-FDTD), and is compared with measured data. The two calculated curves overlap completely, while some difference is observed with the measured data for the highest resonance frequencies.

VI. Conclusion

In this paper, we have presented a new formulation of the FDTD method that reduces computer memory requirements by up to 33%, by eliminating global storage of one electric-field and one magnetic-field component. Using the fact that, in charge-free magnetically homogeneous media, both $\mathbf{D}$ and $\mathbf{H}$ are solenoidal, we linked spatially one $\mathbf{E}$ and one $\mathbf{H}$ component to the other two. This way, the spatially updated components need to be stored only locally. The formulation follows closely that of the FDTD method, as can be seen from the pseudocodes presented for the two- and three-dimensional cases and, therefore, retains most of its simplicity.

Conductors are treated by calculating the charge induced to them by the electromagnetic field, and subsequently using it in the divergence of $\mathbf{D}$. The induced charge is essentially calculated through the continuity equation, although we derived the update equations directly from FDTD since, this way, their implementation is more clear. A treatment very similar to the one used for conductors also yields the equivalent charge induced by the source and, therefore, we were able to extend our method in source regions as well. With these extensions, for conductors and sources, the new formulation becomes general and can be used in the place of regular FDTD, whenever memory savings are critical. In terms of numerical results, it has proven practically equivalent to FDTD for the case examined.

It can be argued that, due to the existence of the three-dimensional matrix $\varepsilon_{i,j,k}$, the actual memory reduction in our formulation is not 33%, but rather 28.6%. However, in applications where memory is the limiting factor, it does not make much sense to use a matrix like that, where very large sub-blocks (corresponding to different dielectric bodies) will contain the same dielectric constant. With an increase in coding complexity, the FDTD method can be formulated to do without $\varepsilon_{i,j,k}$. One simple method is to update the fields inside individual dielectric bodies first and then on the interfaces. Our method also extends rather easily to such an update scheme. On the other hand, the new formulation is computationally more demanding than FDTD. As can be checked by the algorithms, FDTD requires 24 additions and 18 multiplications per cell, while R-FDTD requires 24 additions and 24 multiplications. This increase in multiplications comes from the spatial update of the $\mathbf{E}$-field component through the divergence of $\mathbf{D}$ [see (12)].

In view of the memory savings described above, but also the disadvantages of more computations and a slight increase in coding complexity, the new method we propose should find applications in large electromagnetic problems where memory is the liming factor. For example, we are currently researching the applicability of R-FDTD to wireless channel prediction for indoor spaces, such as residences or offices. Other problems involving scattering from electrically very large bodies should also benefit from this method.

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